V. Conclusions

The proposed method provides a weighting matrix that achieves pole placement for the LQR while making the weighting matrix nonnegative definite by controlling its eigenvalues. The nonnegative definiteness of the weighting matrix ensures desirable properties of the regulator, such as large gain and phase margins. Therefore, the method is a useful pole-assignment method for multi-input state feedback control systems.

References

[1]Kawasaki, N., and Shimemura, E., "Determining Quadratic Weighting Matrices to Locate Poles in a Specified Region," *Automatica*, Vol. 19, No. 5, 1983, pp. 557–560.

[2]Luo, J., and Lan, C. E., "Determination of Weighting Matrices of a Linear Quadratic Regulator," *Journal of Guidance, Control, and Dynamics*, Vol. 18, No. 6, 1995, pp. 1462, 1463.

[3]Sugimoto, K., and Yamamoto, Y., "On Successive Pole Assignment by Linear-Quadratic Optimal Feedbacks," *Linear Algebra and Its Application*, Vol. 122/123/124, Elsevier, New York, 1989, pp. 697–723.

[4]Harvey, C. A., and Stein, G., "Quadratic Weights for Asymptotic Regulator Properties," *IEEE Transactions on Automatic Control*, Vol. AC-23, No. 3, 1978, pp. 378–387.

[5]Ochi, Y., and Kanai, K., "Pole Placement in Optimal Regulator by Continuous Pole-Shifting," *Journal of Guidance, Control, and Dynamics*, Vol. 18, No. 6, 1995, pp. 1253–1258.

Asymptotic Linear Quadratic Control for Lightly Damped Structures

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I. Introduction

LINEAR quadratic control has been used in many capacities for controlled structures problems because of its ease in design and insight it adds into problems such as robust control, structural design, and actuator/sensor selection. Campbell and Crawley[1] developed a set of rules for the design of low-order, robust compensators, called classically rationalized compensators, which utilizes a rigorous examination of the single-input single-output (SISO) linear quadratic Gaussian (LQG) compensator. Wie and Byun[2] compared certain classical design methods with the LQG compensator. And Crawley et al.[3] used both linear quadratic regulator (LQR) and LQG controllers for the preliminary design of a controlled structure.

In experimental applications, however, linear quadratic controllers suffer from important weaknesses such as high order and low robustness.[4] Many design methods attempt to address these concerns within the linear quadratic framework. This Note presents a general form for the LQR and LQG compensators designed for lightly damped structures, and examines the asymptotic properties of these controllers. The asymptotes add insight into how the linear quadratic controllers compensate a flexible system, which can be used to obtain classical design rules motivated by optimal control, improve the order and robustness weaknesses of these controllers, and provide a framework for adding insight to other design methods.

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II. Problem Statement

Consider a system

$$\dot{x} = Ax + B_u u + B_w w$$
$$y = C_y x + v, \qquad z = C_z x \tag{1}$$

in 2 × 2 block modal form, where the ith mode is given by

$$A_i = \begin{bmatrix} 0 & 1 \\ -\omega_i^2 & -2\zeta_i\omega_i \end{bmatrix}, \qquad x_i = \begin{bmatrix} q_i \\ \dot{q}_i \end{bmatrix}, \qquad B_{ui} = \begin{bmatrix} 0 \\ b_{ui} \end{bmatrix}$$

$$B_{wi} = \begin{bmatrix} 0 \\ b_{wi} \end{bmatrix}, \qquad C_{yi} = [c_{yqi} \quad c_{y\dot{q}i}], \qquad C_{zi} = [c_{zqi} \quad c_{z\dot{q}i}]$$

Note that w is a vector of exogenous inputs (disturbances), and v is a vector of sensor noises. The features of a structural system are given: stable, lightly damped, and modally dense.

In transfer function form, the system is given as

$$\begin{bmatrix} z \\ y \end{bmatrix} = \begin{bmatrix} g_{zw} & g_{zu} \\ g_{yw} & g_{yu} \end{bmatrix} \begin{bmatrix} w \\ u \end{bmatrix} = \begin{bmatrix} \dfrac{n_{zw}}{d} & \dfrac{n_{zu}}{d} \\ \dfrac{n_{yw}}{d} & \dfrac{n_{yu}}{d} \end{bmatrix} \begin{bmatrix} w \\ u \end{bmatrix} \tag{2}$$

III. LQR Controller

The LQR controller minimizes a quadratic cost with weightings on the states and inputs:

$$J_{\text{LQR}} = \int_0^\infty \left(x^T R_{xx} x + u^T R_{uu} u\right) \mathrm{d}t \tag{3}$$

In this work, R_{xx} is chosen to penalize the performance and R_{uu} is chosen to weight the inputs equally, or

$$R_{xx} = C_z^T C_z \geq 0, \qquad R_{uu} = \rho \cdot I > 0 \tag{4}$$

where ρ is a positive scalar. Minimization of the LQR cost yields a matrix of optimal gains F for state feedback:

$$u = Fx = -R_{uu}^{-1} B_u^T P x \tag{5}$$

where P is the solution matrix of the algebraic Riccati equation (ARE)

$$PA + A^T P + R_{xx} - P B_u R_{uu}^{-1} B_u^T P = 0 \tag{6}$$

The Kalman filter minimizes the state estimation error where the disturbance and sensor noise are assumed to be zero mean, Gaussian processes that are uncorrelated in time. For this work, the covariances are chosen to be

$$E\{ww^T\} = I, \qquad E\{vv^T\} = \theta \cdot I > 0 \tag{7}$$

where θ is a positive scalar. Note that a dual ARE exists for the Kalman filter.

A. Expensive Control LQR Asymptote

To study the expensive control LQR problem, or as ρ tends to be large, the solution matrix P is expanded in powers of $\sqrt{\rho}$:

$$P = \sqrt{\rho} P_0 + P_1 + (1/\sqrt{\rho}) + \cdots \tag{8}$$

Substituting into Eq. (6), and collecting like powers,

$$\mathcal{O}(\sqrt{\rho}) \quad P_0 A + A^T P_0 = 0 \tag{9}$$

$$\mathcal{O}(1) \quad P_1 A + A^T P_1 + R_{xx} - P_0 B B^T P_0 = 0 \tag{10}$$

This is similar to the expansion by MacMartin,[5] where for an undamped system, the $\mathcal{O}(\sqrt{\rho})$ expensive control LQR gain matrix F is

$$\lim_{\rho\to\infty} F = \frac{1}{\sqrt{\rho}}\sum_i \sqrt{v_i^H R_{xx} v_i}\left[\frac{\left(w_i^H B_u\right)^H}{\left|w_i^H B_u\right|}\right] w_i^H \tag{11}$$

where v_i and w_i are the right and left eigenvectors of the A matrix, respectively, and the H refers to the complex conjugate transpose.

Assuming the system is lightly damped, and because structures are inherently stable, P_0 is identically zero from Eq. 9. Therefore, Eq. (10) reduces to

$$P_1 A + A^T P_1 + R_{xx} = 0 \tag{12}$$

The $\mathcal{O}(1)$ matrix solution P_1 of the ARE for the expensive control problem is the observability gramian. Using state transformation matrices, Gregory[6] showed that the observability gramian can be made diagonal for lightly damped systems and therefore interpreted more easily.

Assuming the system is lightly damped, and the modal spacing is large, P_1 becomes block diagonal, or

$$P_{1i} = \mathrm{diag}\left[\frac{\omega_i}{4\zeta_i}\left(\frac{c_{zqi}^T c_{zqi}}{\omega_i^2} + c_{z\dot{q}i}^T c_{z\dot{q}i}\right), \frac{1}{4\zeta_i\omega_i}\left(\frac{c_{zqi}^T c_{zqi}}{\omega_i^2} + c_{z\dot{q}i}^T c_{z\dot{q}i}\right)\right] \tag{13}$$

where P_{1i} refers to the ith block of P_1 and c_{zqi} and $c_{z\dot{q}i}$ are the position and velocity entries in C_z. Therefore, the gains on the ith position and velocity states are

$$F_{qi} = 0, \qquad F_{\dot{q}i} = -\frac{1}{\rho}\frac{b_{ui}^T}{4\zeta_i\omega_i}\left(\frac{c_{zqi}^T c_{zqi}}{\omega_i^2} + c_{z\dot{q}i}^T c_{z\dot{q}i}\right) \tag{14}$$

The low-gain control effort expended by the LQR controller is to add damping by using velocity feedback. Interpreting the velocity gain, there is a proportional controllability term, an observability term involving both the position and velocity states, and a time constant term, and it is inversely proportional to the scalar weighting ρ.

The velocity gain is also related but not identical to the Hankel singular values[6]

$$\sigma_i^2 = \frac{1}{4\zeta_i\omega_i}\left[b_{ui}b_{ui}^T\left(\frac{c_{zqi}^T c_{zqi}}{\omega_i^2} + c_{z\dot{q}i}^T c_{z\dot{q}i}\right)\right]^{\frac{1}{2}} \tag{15}$$

where σ_i^2 is the Hankel singular value for the ith mode. Therefore, another interpretation of why to retain modes in a model that has large Hankel singular values is that those modes are what LQR tries to control first.

The asymptotes for both lightly damped [Eq. (14)] and undamped [Eq. (11)] systems predict the expensive LQR asymptote quite well. Whereas the asymptote for a lightly damped system is more intuitive, the asymptote for the undamped system has a larger range of applicability. For an undamped, single mode system, Eq. (11) accurately predicts the LQR gain (<5% error) for 0–10% damping. In the lightly damped case (1%), Eq. (14) predicts the LQR gain to the same accuracy for only a 1–1.5% range of damping.

B. Cheap Control LQR Asymptote

The cheap control LQR problem, or as ρ tends to be small, is much more difficult to examine because the ARE contains a singularity. Jameson and O'Malley[7] used singular perturbation theory to find the asymptotic solution of the cheap control LQR problem. The complicated results of this theory, however, make it difficult to add insight into how the controller compensates a structural system.

To gain more understanding of this controller, $C_z(sI - A)^{-1}B_u$ is assumed to be right invertible and minimum phase. For this case, Francis[8] showed that perfect control can be achieved and the matrix solution P of the ARE tends to zero. Kwakernaak and Sivan[9] showed that the cheap control LQR gains for this case are

$$\lim_{\rho\to 0} F = (1/\sqrt{\rho})W_F C_z, \qquad W_F^T W_F = I \tag{16}$$

where W_F is an orthonormal matrix (or ± 1 in the SISO case).

For the case where $C_z(sI - A)^{-1}B_u$ is nonminimum phase, the solution matrix P tends to a constant, which mirrors the unstable system resulting if $P = 0$ were to be used. A treatment of this case can be found in Ref. 10.

Dual low-noise Kalman filter gains are found assuming $C_y(sI - A)^{-1}B_w$ is left invertible and minimum phase:

$$\lim_{\theta\to 0} H = (1/\sqrt{\theta})B_w W_H, \qquad W_H^T W_H = I \tag{17}$$

where W_H is an orthonormal matrix (or ± 1 in the SISO case).

IV. LQG Compensator

The LQG compensator minimizes the $\mathcal{H}_2$ norm from the disturbance to performance variables and is given by

$$\begin{aligned} K(s) &= F(sI - A + B_u F + HC_y)^{-1}H \\ &= F\left[\bar{\Phi}^{-1} + HC_y\right]^{-1}H \end{aligned} \tag{18}$$

where F and H are LQR and Kalman filter gains, respectively, and $\bar{\Phi} = (sI - A + B_u F)^{-1}$.

Using a matrix inversion lemma,

$$(A + BCD)^{-1} = A^{-1} - A^{-1}B(C^{-1} + DA^{-1}B)^{-1}DA^{-1} \tag{19}$$

the following series of simplifications can be made:

$$\begin{aligned} K(s) &= F[\bar{\Phi} - \bar{\Phi}H(I + C_y\bar{\Phi}H)^{-1}C_y\bar{\Phi}]H \\ &= F\bar{\Phi}H[I + C_y\bar{\Phi}H]^{-1} \end{aligned} \tag{20}$$

Splitting the compensator into two parts, and again using the matrix inversion lemma,

$$\begin{aligned} F\bar{\Phi}H &= F\left[\Phi^{-1} + B_u F\right]^{-1}H \\ &= [I + F\Phi B_u]^{-1}F\Phi H \end{aligned} \tag{21}$$

and

$$\begin{aligned} I + C_y\bar{\Phi}H &= I + C_y\left[\Phi^{-1} + B_u F\right]^{-1}H \\ &= I + C_y\Phi\left[I - B_u(I + F\Phi B_u)^{-1}F\Phi\right]H \end{aligned} \tag{22}$$

where $\Phi = (sI - A)^{-1}$. Substituting Eqs. (21) and (22) back into Eq. (20),

$$\begin{aligned} K(s) &= [I + F\Phi B_u]^{-1}F\Phi H \\ &\quad \times \left\{I + C_y\Phi\left[I - B_u(I + F\Phi B_u)^{-1}F\Phi\right]H\right\}^{-1} \\ &= \left[(I + C_y\Phi H)(F\Phi H)^{-1}(I + F\Phi B_u) - C_y\Phi B_u\right]^{-1} \end{aligned} \tag{23}$$

A more intuitive form of the compensator is found by assuming there is one performance, disturbance, input, and output:

$$K(s) = \frac{F\Phi H}{(1 + C_y\Phi H)(1 + F\Phi B_u) - C_y\Phi B_u F\Phi H} \tag{24}$$

Although limited in its applicability, this form can still be used to fully examine the LQG compensator for different topologies of a SISO system and to interpret the controller in a classical manner. To do this, two asymptotes are presented. The low-gain asymptote uses low noise and expensive control; because of its low gain, this asymptote can add insight to the high-frequency region of an LQG controller (after crossover). The high-gain asymptote uses low noise and cheap control; because of its high gain, this asymptote can add insight to the low-frequency region of an LQG controller (before crossover).

A. Low-Gain LQG Asymptote

The low-gain asymptote is defined as the LQG compensator as the sensor noise tends to be small (small θ), and the LQR control tends to be expensive (large ρ). Assuming $C_y\Phi B_w$ is minimum phase, and using the low-noise Kalman filter gain matrix given in Eq. (17), the LQG compensator reduces to

$$\lim_{\theta\to 0} K(s) = \frac{F\Phi B_w}{C_y\Phi B_w + C_y\Phi B_w F\Phi B_u - C_y\Phi B_u F\Phi B_w} \tag{25}$$

This compensator design is a generalized result of the loop transfer recovery method.[11]

Using the expensive control LQR gain matrix F as derived in Eq. (14), then

$$\lim_{\substack{\theta\to 0\\ \rho\to\infty}} K(s) = \frac{F\Phi B_w}{C_y\Phi B_w} \tag{26}$$

Equation (26) is a ratio of two transfer functions with identical denominators (same poles for each system). Therefore, the compensator further simplifies to

$$\lim_{\substack{\theta\to 0\\ \rho\to\infty}} K(s) = \frac{\sum_{i=1}^{n} F_{\dot{q}i} b_{wi} s \prod_{\substack{j=1\\ j\neq i}}^{n} \left(s^2 + 2\zeta_i\omega_i s + \omega_i^2\right)}{n_{yw}(s)} \tag{27}$$

where $F_{\dot{q}i}$ are the LQR gains on the velocity states, ζ_i and ω_i are the ith damping ratio and frequency, and $n_{yw}(s)$ is the numerator of the disturbance-output transfer function.

The numerator of this compensator is a weighted mixture of the poles of the system. Many times, the system has a dominant mode as defined by the largest term $F_{\dot{q}i} b_{wi}$. Ordering this mode first, the compensator simplifies to

$$K_{\mathrm{LG}}(s) = \lim_{\substack{\theta\to 0\\ \rho\to\infty}} K(s) = \frac{F_{\dot{q}1} b_{w1} s \prod_{i=2}^{n} \left(s^2 + 2\zeta_i\omega_i s + \omega_i^2\right)}{n_{yw}} \tag{28}$$

This compensator is the SISO low-gain LQG asymptote. Note that an analogous asymptote can be defined for high noise (large θ) and cheap control (small ρ).

The asymptote is a low-gain, rate feedback inversion of the disturbance-output transfer function, except for the dominant mode. Therefore, in addition to the obvious influences of the g_{yu} (stability) and g_{zw} (performance) transfer functions, linear quadratic control design is heavily influenced by the g_{yw} and g_{zu} transfer functions.

B. High-Gain LQG Asymptote

The high-gain asymptote is defined as the LQG compensator as the sensor noise tends to be small (small θ) and the LQR control tends to be cheap (small ρ). Assuming both $C_z\Phi B_u$ and $C_y\Phi B_w$ are minimum phase, and using the cheap control and low-noise asymptotes given in Eqs. (16) and (17), the LQG compensator reduces to

$$K_{\mathrm{HG}}(s) = \lim_{\substack{\theta\to 0\\ \rho\to 0}} K(s) = \frac{\pm g_{zw}}{\pm\sqrt{\rho}\, g_{yw} \pm \sqrt{\theta}\, g_{zu} \pm (g_{zu}g_{yw} - g_{yu}g_{zw})} \tag{29}$$

This compensator is the SISO high-gain LQG asymptote.

Interpretation of this asymptote is easier when considering a closed-loop system with compensator $K(s)$ for the simplified single performance, output, disturbance, and input case. The disturbance to performance transfer function is given by

$$(g_{zw})_{\mathrm{CL}} = \frac{g_{zw} + (g_{zw}g_{yu} - g_{zu}g_{yw})K}{1 + g_{yu}K} \tag{30}$$

The term $(g_{zw}g_{yu} - g_{zu}g_{yw})$ is the determinate of the transfer function matrix given in Eq. (2). If this determinate is 0 (or small), the asymptote (and low-frequency LQG controller) achieves performance using very high gain. If this determinate is large (the most common system) the asymptote achieves performance by creating a subtraction in the numerator of the closed-loop disturbance to performance transfer function. Because this type of control is quite nonrobust, it is unlikely that the input-output pair would achieve very good performance. This insight can be used to choose specific input(s) and output(s) for controlling given disturbance(s) to performance(s).

V. Conclusions

A thorough examination of the properties of the LQG compensator has been conducted. The expensive LQR gains for lightly damped structures are derived, which are nonzero on the rate states only and are dependent on the controllability, observability, and time constant of each particular mode.

A useful form of the SISO LQG compensator is derived, which enables the examination of the asymptotic properties of the LQG compensator. Assuming the disturbance-output transfer function is minimum phase and contains a single dominant mode, for low noise and expensive control, the SISO LQG compensator converges to a low-gain, rate feedback inversion of the disturbance-output transfer function, except for the dominant mode. This low-gain asymptote represents the LQG controller at high frequency (after crossover).

Assuming both the disturbance-output and input-performance transfer functions are minimum phase, the SISO high-gain LQG asymptote is derived for low noise and cheap control and represents the LQG controller at low frequency (before crossover). When the transfer function matrix is singular, the asymptote is a high-gain compensator. When the transfer function matrix is full rank, the asymptote converges to a constant and creates a nonrobust subtraction in the numerator of the closed-loop disturbance-performance transfer function, an insight that can be used for actuator/sensor selection.

References

[1]Campbell, M. E., and Crawley, E. F., "Classically Rationalized Low Order Robust Structural Controllers," *Proceedings of the AIAA 35th Structures, Structural Dynamics and Materials Conference* (Hilton Head, SC), AIAA, Washington, DC, 1994, pp. 1923–1935 (AIAA Paper 94-1564).

[2]Wie, B., and Byun, K.-W., "New Generalized Structural Filtering Concept for Active Vibration Control Synthesis," *Journal of Guidance, Control, and Dynamics*, Vol. 12, No. 2, 1989, pp. 147–154.

[3]Crawley, E. F., Masters, B. P., and Hyde, T. T., "Conceptual Design Methodology for High Performance Dynamic Structures," *Proceedings of the AIAA Structures, Structural Dynamics, and Materials Conference* (New Orleans, LA), AIAA, Washington, DC, 1995, pp. 2060–2073 (AIAA Paper 95-1407).

[4]Doyle, J. C., "Guaranteed Margins for LQG Regulators," *IEEE Transactions on Automatic Control*, Vol. AC-23, No. 4, 1978, pp. 756, 757.

[5]MacMartin, D. G., "An $\mathcal{H}_\infty$ Power Flow Approach to Control of Uncertain Structures," M.S. Thesis, Dept. of Aeronautics and Astronautics, Massachusetts Inst. of Technology, Cambridge, MA, Feb. 1990.

[6]Gregory, C. Z., Jr., "Reduction of Large Flexible Spacecraft Models Using Internal Balancing Theory," *Journal of Guidance, Control, and Dynamics*, Vol. 7, No. 6, 1984, pp. 725–732.

[7]Jameson, A., and O'Malley, R. E., Jr., "Cheap Control of the Time Invariant Regulator," *Applied Mathematics and Optimization*, Vol. 1, No. 4, 1975, pp. 337–354.

[8]Francis, B. A., "The Optimal Linear-Quadratic Time-Invariant Regulator with Cheap Control," *IEEE Transactions on Automatic Control*, Vol. AC-24, No. 4, 1979, pp. 616–621.

[9]Kwakernaak, H., and Sivan, R., "The Maximally Achievable Accuracy of Linear Optimal Regulators and Linear Optimal Filters," *IEEE Transactions on Automatic Control*, Vol. AC-17, No. 1, 1972, pp. 79–86.

[10]Saberi, A., and Sannuti, P., "Cheap and Singular Controls for Linear Quadratic Regulators," *IEEE Transactions on Automatic Control*, Vol. AC-32, 1987, pp. 208–219.

[11]Doyle, J. C., and Stein, G., "Multivariable System Design: Concepts for a Classical/Modern Synthesis," *IEEE Transactions on Automatic Control*, Vol. AC-26, No. 1, 1981, pp. 4–16.

Real-Time Optimal State Feedback Control for Tethered Subsatellite System

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Introduction

ALTHOUGH many papers are devoted to the study of control problems of the tethered subsatellite systems, only a few papers[1–4] treat the control problems as optimization problems. Bainum and Kumar[1] introduced an optimal-control law based on an application of the linear quadratic regulator (LQR) to the tethered satellite system. They apply the tension control law, employing feedback of tether length, in-plane angle, their rates, and the commanded length for the deployment and retrieval and stationkeeping of the subsatellite; the performance index for the optimization problems is set to correct deviations from the instantaneous equilibrium state at each time. No and Cochran[2] show that the LQR can be applied to control stationkeeping and maneuvering of the tethered subsatellite by utilizing the atmospheric aerodynamics, and the performance index for the optimization problem is set to correct deviations from the reference trajectory and to converge into the desired state at the final time. Netzer and Kane[3] obtain an optimal-length law of the tethered subsatellite system with the performance index involving penalties on the terminal values of the states as well as on the states and controllers throughout the maneuver, and the optimal solution is used as a nominal path to follow. They show that the LQR can be applied to derive tracking-type feedback control law to decrease the deviation of the subsatellite from the nominal path by using the thrusters. On the assumption that the control force is only the tether tension and that no control force or energy dissipation exists for motion perpendicular to the tether line, Fujii and Anazawa[4] obtain an optimal path in the sense that the time integral of squared tension plus squared in-plane angle is the performance index with inequality constraints on the control force. The Lyapunov approach is applied to derive the tracking-type feedback control law to follow the optimal path in Ref. 4. It is generally difficult to obtain the optimal path by solving the two-point boundary-value problem. To overcome this difficulty, Ohtsuka and Fujii[5] adopted the stabilized continuation method,[6] which converts the two-point boundary-value problem into the initial-value problem. Combining this method with the receding horizon control method,[7] they have succeeded in developing the real-time optimal state feedback controller.[8] This Note applies this real-time optimal state feedback control to the deployment and retrieval control problem of the tethered subsatellite and to determine the effective application of the control.

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Tethered Subsatellite System

A tethered subsatellite connected to the Shuttle is illustrated in Fig. 1. The center of attraction is denoted by O and the center of mass of the Shuttle by C. The orthogonal axes X and Y are defined along OC and along the orbital velocity vector, respectively, both originating at C. The parameters m, l, and Θ denote mass of the subsatellite, length of the tether, and rotational angle of the subsatellite in the orbital plane, respectively.

In this study, the following assumptions are made:

1) The tether has no mass, and thus its flexibility is ignored.

2) The mass of the subsatellite is sufficiently small with respect to that of the Shuttle that C always remains in its nominal orbit.

3) The external force affecting the motion is only the gravitational force caused by O. The orbit is circular, and only motion in the orbital plane is considered.

4) The control force acts only along the tether through tension $\tilde{T}$, and no control force or energy dissipation exists for motion perpendicular to the tether line.

On the above assumptions, the dimensionless equations of motion are obtained as follows:

$$\Lambda'' - \Lambda\Theta'^2 - 2\Lambda\Theta' - 3\Lambda\cos^2\Theta = -\hat{T} \tag{1}$$

$$\Theta'' + 2(\Lambda'/\Lambda)\Theta' + 3\sin\Theta\cos\Theta + 2(\Lambda'/\Lambda) = 0 \tag{2}$$

where $()' = \mathrm{d}()/\mathrm{d}\tau$, where $\tau = \Omega t$, t is time, and Ω is the angular velocity of the Shuttle in its orbit; $\Lambda = l/L$, where L is the desired length for the deployment; and $\hat{T} = \tilde{T}/(m\Omega^2 L)$.

Real-Time Optimal State Feedback Control for the Tethered Subsatellite System

Solution of the optimal control problem, in general, is obtained by solving the two-point boundary-value problem, which consists of Euler–Lagrange equations derived from the stationary condition of augmented performance index with multiplier functions.[9] Numerical calculation is necessary to solve the two-point boundary-value problem with iterative process because it is difficult to obtain an optimal solution analytically. Several methods have been proposed to solve the optimal control problem.[10–12] These methods require a suitable candidate of the initial guess because the accuracy of convergence into the optimal solution depends on the initial guess. Such a solution is, however, difficult to find, and an unsuitable solution makes it difficult to satisfy the boundary conditions. To overcome this difficulty, the two-point boundary-value problem is converted into the initial-value problem by the continuation method[5,6] in this Note. The following performance index is considered for the tethered subsatellite system:

$$J = \frac{1}{2}c_x(x - x_f)\Big|^2_{\tau = T_f} + \frac{1}{2}R\int_0^{T_f}\hat{T}^2\,\mathrm{d}\tau \tag{3}$$

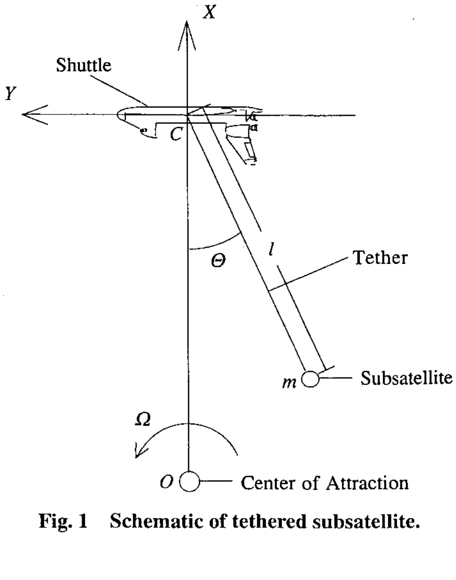

Fig. 1 Schematic of tethered subsatellite.